

# Separating Different Models from Measuring $\alpha$ , $\beta$ , $\gamma$

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## Abstract

New physics effects on the  $B^0 - \bar{B}^0$  mixing and  $B$ -decay amplitudes are discussed. By a combined analysis, the models of new physics can be partially distinguished. It is emphasized that the extraction of unitarity angles  $\beta$  and  $\gamma$  through rare decay  $K \rightarrow \pi\nu\bar{\nu}$  and charged  $B$  decay  $B^\pm \rightarrow DK^\pm$  respectively is not likely to be affected by new physics. Such an observation could be used to distinguish the new physics effects from different models. For instance, the top quark two-Higgs doublet model can be easily separated from those models without new phase in  $B$ -decay amplitudes, and can be further distinguished from the most general two Higgs doublet model by its absence of new phase in the  $B^0 - \bar{B}^0$  mixing.

## I. INTRODUCTION

In the standard model(SM) with  $SU(2)_L \times U(1)_Y$  gauge symmetry and three generation of fermions. The only source of CP asymmetry is a non-zero complex phase in the Cabibbo-Kobayash-Maskawa(CKM) matrix. Although SM has been proved to be very successful in phenomenology, its accommodation of CP violation through complex CKM matrix elements has not been seriously tested experimentally. At present, CP violation is one of the least understood issues in particle physics, and is very promising in the search of indications of new physics.

Assuming unitarity of CKM matrix, the phase information of the matrix can be displayed elegantly by a set of triangles, called Unitarity triangles(UTs) . The central topic is therefore the determination of the angles of those triangles. In the past years, much efforts have been made in the neutral-Kaon system as well as in  $B_d^0$  and  $B_s^0$  mixings. However, due to large theoretical uncertainties, our current knowledge of those angles is still very poor. It is expected that in the up coming B-factories, the measurements of time dependent CP asymmetries in  $B^0$  decays into CP eigenstates will greatly reduce the hadronic uncertainties and obtain the precise value of the angles of UT with  $b$  and  $d$  quarks. If those angles are precisely determined, any deviation from the SM predictions will clearly signal new physics beyond the SM.

Supposing all the angles  $(\alpha, \beta, \gamma)$  are completely determined through independent measurements, following the analysis in reference [1], there are three distinct ways in which new physics can show up in the measurements of CP asymmetry, they are:

$$1) \alpha + \beta + \gamma \neq \pi,$$

2) $\alpha + \beta + \gamma = \pi$ , but the value of  $\alpha, \beta$  and  $\gamma$  do not agree with the SM predictions.  
 3) $\alpha + \beta + \gamma = \pi$ ,  $\alpha, \beta$  and  $\gamma$  are consistent with the SM, but measurement of the angles are inconsistent with the measurements of the side of the UT.

If any one of these three cases really happens in the future B- factory, the new physics will be established. However, this only tells us that new physics exactly exists. We still don't know what kind of new physics is responsible, since there are variety of models of new physics which can affect the value of angles in the same way. It is of great importance to distinguish different models of new physics from the experiment.

The problem of distinguishing various models of new physics has been discussed in [1,2]. It may involve many models of new physics such as supersymmetric models [3], multi-higgs [4] without flavor changing neutral scalars interactions or general two-higgs-doublet model with flavor changing neutral scalars interactions (S2HDM) [5,6], left-right symmetric model [7], Z-mediated flavor changing neutral currents(FCNC) [8] and fourth generation [9]. The conclusion is that by comparing their contributions to  $B^0 - \bar{B}^0$  mixings and rare leptonic  $B$  decays, these models can be partially distinguished. If new physics is founded to be the case 1) or 2) as mentioned above. This would indicate that new physics is probably to be the S2HDM, fourth-generation or Z-mediated FCNC. If new physics is founded through the case 3), the new physics is likely to be two-Higgs doublet model without flavor changing neutral scalar interactions (such as Model 1 or Model 2) or minimum supersymmetry model. This method is very useful, but still not sufficient to distinguish each of the models especially when several models have similar effect on the extraction of those unitarity angles. In this paper, We show that the combination of analysis on  $B^0 - \bar{B}^0$  mixing and  $B$ -decay amplitude via time dependent measurement of CP asymmetry [10] is also an efficient way in separating different models. As an example , the top quark two-Higgs doublet model(T2HDM) recently discussed in [11] can be distinguished not only from a large number of model without CP asymmetry in  $B$ -decay amplitude, but also from the S2HDM by their different behavior in  $B^0 - \bar{B}^0$  mixing. The paper is organized as follows: In section **II** , we present the basic formulas on distinguishing different models of new physics by considering their difference in  $B^0 - \bar{B}^0$  mixing and  $B$ -decay amplitude, some features of the S2HDM and T2HDM as well as their influences on the determination of angles  $\alpha, \beta$  and  $\gamma$  are discussed in section **III**. The conclusions are presented in section **IV**.

## II. BASIC FORMULAS

From the theoretical point of view, there are two basic ways in which new physics can enter the extraction of angles  $\alpha, \beta$  and  $\gamma$ . One way is via  $B^0 - \bar{B}^0$  mixings, the other is via  $B$  decay amplitudes which is mainly through hadronic penguin diagrams, but in some models it may be also through tree diagrams, such as the S2HDM and T2HDM. That will be discussed in detail below.

If  $B^0 - \bar{B}^0$  mixing is affected by new physics, for example, from additional heavy particles in the loop instead of  $W$ -boson, the angle  $\beta$  measured in the process  $B_d^0 \rightarrow J/\psi K_S$  can be largely modified. In the SM the time dependent asymmetry is given as follows:

$$Im\lambda = \left(\frac{q}{p}\right)_{B_d} \left(\frac{\bar{A}}{A}\right) \left(\frac{p}{q}\right)_K = -\sin 2\beta, \quad (2.1)$$

where the term in the first bracket is from  $B^0 - \bar{B}^0$  mixing which has the value of  $V_{td}V_{tb}^*/V_{td}^*V_{tb}$  in the SM,  $A(\bar{A})$  is the amplitude of  $b \rightarrow c(\bar{c}s)(\bar{b} \rightarrow \bar{c}(c\bar{s}))$  subprocess, the term in the last bracket is from  $K^0 - \bar{K}^0$  mixing since  $K_S$  is involved in the final state. Without losing generality, the new physics can affect all these three quantities. Let us denote  $\phi_{mix}^{B_d}$ ,  $\phi_A$  and  $\phi_{mix}^K$  the new phase from new physics in  $B_d^0$  mixing,  $B$ -decay amplitude, and  $K^0$  mixing respectively. Then the experiment will measure  $\beta_{exp}$  instead of  $\beta_{SM}$  as:

$$\beta_{exp} = \beta_{SM} + \phi_{mix}^{B_d} + \phi_A(b \rightarrow c) + \phi_{mix}^K \quad (2.2)$$

where the process  $b \rightarrow c$  indicated in the bracket is a tree level transition. The angle  $\alpha$  can be measured from  $B_d^0$  decay to  $\pi^+\pi^-$ . In this channel there are also contributions from penguin diagrams with different strong phase, this can be eliminated by using the isospin analysis [12]. As it was pointed out in [2] and also emphasized by many authors [13–15], if the angle  $\alpha$  is measured through the decay channel  $B_d^0 \rightarrow \pi^+\pi^-$ , then the new physics effect will give contributions with an opposite sign as follows:

$$\alpha_{exp} = \pi - \beta_{SM} - \gamma_{SM} - \phi_{mix}^{B_d} - \phi_A(b \rightarrow u) \quad (2.3)$$

Here the process is  $(b \rightarrow u)$  as  $B_d^0 \rightarrow \pi^+\pi^-$  is dominated by  $b \rightarrow u$  transition. Consequently, the new phase  $\phi_{mix}^{B_d}$  in  $B^0 - \bar{B}^0$  mixing *cancels* each other in the sum  $\alpha_{exp} + \beta_{exp}$ . When the new phase from amplitudes and  $K$ -meson mixing are negligible small( these happens in many models ), the sum  $\alpha + \beta$  will remain unchanged. If the angle  $\gamma$  is determined through charged  $B$  decay  $B^\pm \rightarrow DK^\pm$ , since  $B^0$ -mixing is absent and the FCNC will not be involved in this channel, its value can hardly be modified by new physics, so  $\gamma_{exp}$  is likely to be unchanged and equals  $\gamma_{SM}$ . Thus the sum

$$\alpha_{exp} + \beta_{exp} + \gamma_{exp} = \pi \quad (2.4)$$

still holds as in the case of SM.

The new phase  $\phi_{mix}^K$  in  $K$ -mixing is often thought to be small, this is because the extremely small values of  $\Delta m_K$  and  $\epsilon_K$  impose a very strong constraint on the contributions to  $K^0 - \bar{K}^0$  mixing from new physics. Thus as a consequence, the new physics can not produce a relative large value of  $\phi_{mix}^K$ . It is a possibility that by observing the violation of equation:

$$Im\lambda(B_d \rightarrow D^+D^-) = Im\lambda(B_d \rightarrow J/\psi K_S) \quad (2.5)$$

or

$$Im\lambda(B_d \rightarrow D^+D^-) = Im\lambda(B_d \rightarrow \phi K_S) \quad (2.6)$$

One is able to probe the new physics in  $K^0 - \bar{K}^0$  mixing [15]. In the following discussion we always make the assumption that the new phase in  $K^0$  mixing is negligible.

Although the effect of decay amplitude  $\phi_A$  is always thought to be small, its importance on signaling new physics should not be neglected. As being stressed in reference [16], the effects of new physics in decay amplitudes are manifestly non-universal, because they strongly depend on the specific process and decay channel under consideration. On the other hand,

the effects on  $B^0 - \bar{B}^0$  mixing are almost insensitive to the decay modes. Since in general  $\phi_A(b \rightarrow c) \neq \phi_A(b \rightarrow u)$ , in the condition that  $\phi_{mix}^K$  is zero, the equation(2.4) becomes

$$\alpha_{exp} + \beta_{exp} + \gamma_{exp} = \pi + \phi_A(b \rightarrow c) - \phi_A(b \rightarrow u). \quad (2.7)$$

this will be a clear signal of new physics from decay amplitude.

By considering whether the equation(2.4) holds, the models of new physics can be cataloged into two classes, i.e. models with or without new phase in decay amplitude. A large number of models such as the 2HDM of types **I** and **II**, left-right symmetric, and the minimum supersymmetric model, fourth generation and Z-mediated flavor changing neutral currents(FCNC) fall into the first class [1], whereas the S2HDM and T2HDM as well as some other models fall into the second class. Therefore the models can be partially distinguished in this way.

Another useful information is the new phase in  $B^0 - \bar{B}^0$  mixing. However, due to their cancellation in the sum  $\alpha + \beta$  it can not be extracted directly. If one looks at the  $B_d^0$  decay to CP eigenstates such as  $J/\psi K_S$ ,  $D^+ D^-$  or  $\phi K_S$  the new phases from  $B^0$ -mixing and from  $B$  decay amplitude may mix with each other, and the final result is the sum of these two kind of contributions. Thus the extraction of pure new phase from  $B^0 - \bar{B}^0$  mixing in  $B_d^0 \rightarrow J/\psi K_S$  becomes difficult.

This situation can be simplified if one of the decay amplitudes  $\phi_A(b \rightarrow c)$  and  $\phi_A(b \rightarrow u)$  is negligible small. For example,  $\phi_A(b \rightarrow u) \sim 0$  while  $\phi_A(b \rightarrow c)$  is obviously non-zero ( this happens in the case of the S2HDM and T2HDM which is under consideration of this paper ), thus equation(2.7) becomes

$$\alpha_{exp} + \beta_{exp} + \gamma_{exp} = \pi + \phi_A(b \rightarrow c). \quad (2.8)$$

Thus  $\phi_A(b \rightarrow c)$  can be easily obtained by measuring the sum of  $\alpha$ ,  $\beta$ , and  $\gamma$ . Substituting the value of  $\phi_A(b \rightarrow c)$  into equation(2.2), the phase  $\phi_{mix}^{B_d}$  can be fixed. In doing this, we need to know the SM prediction of  $\beta_{SM}$  since many decay channels can be seriously polluted by new physics, one should carefully choose some processes which are not likely to be modified. One way is to use the value of  $|V_{cb}|$ ,  $|V_{ub}|/|V_{cb}|$  from semileptonic  $B$  decays  $b \rightarrow c(u) l \bar{\nu}_l$  and  $\gamma$  from  $B^\pm \rightarrow D K^\pm$  [17]. These three quantities correspond to two sides and one angle between the two sides in UT. Thus the whole triangle including angle  $\beta$  can be completely determined. The shortcoming here is that the prediction of  $|V_{ub}|/|V_{cb}|$  is model dependent and suffer a large theoretical uncertainties.

An alternative way to extract  $\beta$  is via rare  $K$  decay [18–20]  $K \rightarrow \pi \nu \bar{\nu}$ . The branching ratio of decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  is given as follows:

$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa \left[ \left( \frac{Im \lambda_t}{\lambda^5} X(x_t) \right)^2 + \left( \frac{Re \lambda_c}{\lambda} P_0(K^+) + \frac{Re \lambda_t}{\lambda^5} X(x_t) \right)^2 \right] \quad (2.9)$$

with

$$\kappa = \frac{3\alpha^2 B(k^+ \rightarrow \pi^0 e^+ \nu)}{2\pi^2 \sin^4 \theta_W} \lambda^8 = 4.64 \times 10^{-11} \quad (2.10)$$

where  $X(x_i)$  is an integral function given in [18],  $x_t = m_t^2/m_W^2$ ,  $\lambda_i = V_{is}^* V_{id}$  and  $\lambda = |V_{us}| \sim 0.22$ . The function  $P_0(K^+)$  has the form  $P_0(K^+) = (2X_{NL}^e/3 + X_{NL}^\tau/3)/\lambda^4$ . By combining the branching ratio of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ :

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{Im\lambda_t}{\lambda^5} X(x_t) \right)^2 \quad (2.11)$$

$$\kappa_L = \kappa \frac{\tau(K_L)}{\tau(K^+)} = 1.94 \times 10^{-10} \quad (2.12)$$

one can find:

$$Im\lambda_t = \lambda^5 \frac{\sqrt{B_2}}{X(x_t)} \quad Re\lambda_t = -\lambda^5 \frac{\frac{Re\lambda_c}{\lambda} P_0(K^+) + \sqrt{B_1 - B_2}}{X(x_t)} \quad (2.13)$$

where  $B_1$  and  $B_2$  are the reduced branching ratios with  $B_1 = B(K^+ \rightarrow \pi^+ \nu \bar{\nu})/4.64 \times 10^{-11}$  and  $B_2 = B(K^+ \rightarrow \pi^0 \nu \bar{\nu})/1.94 \times 10^{-10}$ .

Using the standard parameterization of the CKM matrix, the angle  $\beta$  can be determined by

$$\sin 2\beta = \frac{2r_s}{1 + r_s^2} \quad (2.14)$$

with  $r_s = (1 - \bar{\rho})/\bar{\eta}$ . The parameter  $\bar{\rho}$  and  $\bar{\eta}$  is given as follows:

$$\bar{\rho} = \frac{\sqrt{1 + 4s_{12}c_{12}Re\lambda_t/s_{23}^2 - (2s_{12}c_{12}Im\lambda_t/s_{23}^2)^2} - 1 + 2s_{12}^2}{2c_{23}^2s_{12}^2} \quad (2.15)$$

$$\bar{\eta} = \frac{c_{12}Im\lambda_t}{s_{12}c_{23}^2s_{23}^2} \quad (2.16)$$

It is well known that the rare decays  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  can be calculated with smaller theoretical uncertainties. These uncertainties can be further reduced in the next-to-leading order QCD corrections [21–23]. As a result, the measurement of both two decays with an error of  $\pm 10\%$  will yield  $\sin 2\beta$  with an accuracy comparable to the determination from CP asymmetry in  $B$ -decays prior to LHC [18].

Here we emphasize that these channels are not likely to be affected by new physics models, especially, the models with FCNC. This is because the couplings between fermions and additional scalars which often present in the models of new physics are proportional to the fermion mass, the decay involving leptons in the final states will greatly suppress the tree level contributions from those scalars. Although there may be significant new physics contributions to  $Z$ -penguin diagrams, the angle  $\beta$  will remain unchanged if the new physics do not carry additional new phase. This happens in many models with additional Higgs bosons, such as 2HDM of type **I**, and type **II**, minimum supersymmetric models, et.al.

If  $\beta_{SM}$  is extracted in this way, it is then possible to study the behavior of different models in  $B^0 - \bar{B}^0$  mixing independent of  $B$ -decay amplitudes. In general, different models have different behavior in  $B^0$  mixings and decay amplitudes. It is therefore possible to identify the models by combining the analysis of these two aspects. Following this strategy, the T2HDM can be distinguished not only from those models without new phase in  $B$  decay amplitudes, but also from the S2HDM by its absence of contribution in  $B^0$  mixing. This will be further discussed in the next section.

### III. ON S2HDM AND T2HDM

Let us briefly present some important prospects of T2HDM. The Lagrangian of T2HDM is as follows:

$$\mathcal{L}_Y = -\bar{L}_L \phi_1 E l_R - \bar{Q}_L \phi_1 F d_R - \bar{Q}_L \tilde{\phi}_1 G I^{(1)} u_R - \bar{Q}_L \tilde{\phi}_2 G I^{(2)} u_R + h.c, \quad (3.1)$$

Where  $\bar{L}_L$  and  $\bar{Q}_L$  are the ordinary left-handed lepton and quark doublets,  $u_R$  and  $d_R$  are right-handed singlet quarks,  $\phi_1$  and  $\phi_2$  are two Higgs doublets with  $\tilde{\phi}_i = i\sigma^2\phi_i^*$  and  $E, F, G$  are Yukawa coupling matrix.  $I^{(1)}$  and  $I^{(2)}$  are two diagonal matrix with  $I_{ij}^{(1)} = \delta_{ij}$  ( $i, j = 1, 2$ ) and  $I_{ij}^{(2)} = \delta_{ij}$  ( $i = j = 3$ ).

Comparing with other quarks, the top quark is in a special status in this model, i.e. Only  $\phi_2$  couples to  $t_R$ . Let the vacuum expectation value(VEV) of two Higgs fields to be  $v_1/\sqrt{2}$  and  $v_2 e^{i\delta}/\sqrt{2}$  respectively. If we choose the ratio between two VEVs  $\tan\beta = |v_2|/|v_1|$  to be large ( $\tan\beta$  is close to  $m_t/m_b$ ), the large mass of top quark can be naturally explained. That is the motivation of proposing this model.

The charged quark-Higgs Yukawa interaction in this model reads

$$\begin{aligned} \mathcal{L}^C = & -2\sqrt{2}G_F[-\bar{u}_L^i V_{ij} m_{d_j} d_R^j \tan\beta \\ & + \bar{u}_R^i m_u V_{ij} d_R^j \tan\beta + \bar{u}_R^i \Sigma_{ij'}^\dagger V_{j'j} d_L^j (\tan\beta + \cot\beta)] H^+ + H.c, \end{aligned} \quad (3.2)$$

where  $m_i$  are the quark mass eigenstates,  $V$  is the usual CKM matrix. The matrix  $\Sigma$  can be parameterized as follows:

$$\Sigma = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_c \epsilon_{ct}^2 |\xi|^2 & m_c \epsilon_{ct} \xi^* \sqrt{1 - |\epsilon_{ct} \xi|^2} \\ 0 & m_c \xi \sqrt{1 - |\epsilon_{ct} \xi|^2} & m_t (1 - |\epsilon_{ct} \xi|^2) \end{pmatrix} \quad (3.3)$$

Since there exist none zero off-diagonal elements in  $\Sigma$  matrix, this model may lead to flavor changing neutral currents, but only in up-type quarks. Therefore it will enhance the  $D^0 - \bar{D}^0$  mixing provided that  $\tan\beta$  is large [24,11]. On the other hand, it has little effect on  $K^0 - \bar{K}^0$  mixing and  $B^0 - \bar{B}^0$  mixing since these mesons contain down-type quarks. Another distinct feature is that the off-diagonal element  $\Sigma_{32}$  can be quit large, thus the model generally has a very large couplings for the vertex  $\bar{b}cH^-$  or  $\bar{t}cH^0$ .

Let us turn to a brief discussion on the S2HDM. This model can be obtained if we abandon the discrete symmetry which is often imposed on the Lagrangian of two Higgs doublet model [25] and replace it with an approximate global U(1) family symmetry [5,6]. The point is that the smallness of the off-diagonal terms in the CKM matrix suggests that violation of flavor symmetry are specified by small parameters. It then turns out that reasonable choices for these small parameters combined with the natural smallness of Higgs boson couplings allows one to meet the constraint on flavor changing neutral scalar exchange. Since there are no discrete symmetries, many new sources of CP asymmetry can arise from its lagrangian [6]. Therefore, this model can affect the measurement of the angles of UT in many different ways. For example the possible large effects on  $B^0 - \bar{B}^0$  mixing [24], weak transition  $t \rightarrow c$  and large CP asymmetry in  $b \rightarrow s\gamma$  [27] have been investigated.

The Lagrangian of S2HDM has the form:

$$\mathcal{L}_Y = \bar{Q}^i \Gamma_{1,ij}^U U_{Rj} \phi_1 + \bar{Q}^i \Gamma_{1,ij}^D D_{Rj} \tilde{\phi}_1 + \bar{Q}^i \Gamma_{2,ij}^U U_{Rj} \phi_2 + \bar{Q}^i \Gamma_{2,ij}^D D_{Rj} \tilde{\phi}_2 + h.c. \quad (3.4)$$

After a rotation into quark mass eigenstates, it can be rewritten as [6]:

$$\mathcal{L}_Y = (L_1 + L_2) \cdot (\sqrt{2} G_F)^{1/2} \quad (3.5)$$

with

$$\begin{aligned} \mathcal{L}_1 = & \sqrt{2} (H^+ \sum_{i,j}^3 \xi_{d_j} m_{d_j} V_{ij} \bar{u}_L^i d_R^j - H^- \sum_{i,j}^3 \xi_{u_j} m_{u_j} V_{ij}^\dagger \bar{d}_L^i u_R^j) \\ & + H^0 \sum_i^3 (m_{u_i} \bar{u}_L^i u_R^i + m_{d_i} \bar{d}_L^i d_R^i) \\ & + (R + iI) \sum_i^3 \xi_{d_i} m_{d_i} \bar{d}_L^i d_R^i + (R - iI) \sum_i^3 \xi_{u_i} m_{u_i} \bar{u}_L^i u_R^i + H.c. \end{aligned} \quad (3.6)$$

$$\begin{aligned} \mathcal{L}_2 = & \sqrt{2} (H^+ \sum_{i,j' \neq j}^3 V_{ij'} \mu_{j'j}^d \bar{u}_L^i d_R^j - H^- \sum_{i,j' \neq j}^3 V_{ij'}^\dagger \mu_{j'j}^u \bar{d}_L^i u_R^j) \\ & + (R + iI) \sum_{i \neq j}^3 \mu_{ij}^d \bar{d}_L^i d_R^j + (R - iI) \sum_{i \neq j}^3 \mu_{ij}^u \bar{u}_L^i u_R^j + H.c. \end{aligned} \quad (3.7)$$

Where the factors  $\xi_{d_j} m_{d_j}$  arise primarily from diagonal elements of  $\Gamma_1$  and  $\Gamma_2$  whereas the factors  $\mu_{j'j}^d$  arise from the small off-diagonal elements.

By abandoning the discrete symmetry, this model obtains rich sources of CP violation. They can be classified into four major types [6]: (1) The induced CKM matrix. (2) The phases in the factors  $\xi_{f_i}$  provide CP violation in the charged-Higgs exchange processes, which are independent of the CKM phase. (3) The phases in the factors  $\mu_{ij}^f$ . These yield CP violation in flavor changing neutral scalar interaction. (4) the phase from the mixing matrix of the three neutral Higgs scalars

Although these two models have some similar behavior in CP asymmetry, there still exist several subtle differences between them.

First, in the T2HDM the coupling between quarks and Higgs boson is determined by only parameters  $\tan \beta$  and  $\xi$ . On the other hand such couplings in the S2HDM are flavor dependent. So that the latter has more freedom in fitting the experimental data.

Second, there is no complex phase in the diagonal Yukawa couplings in T2HDM. This means that there is no CP asymmetry from charged Higgs exchange in  $t \rightarrow b$  transition. So it will result in a small CP asymmetry in the decay  $b \rightarrow s\gamma$ , which is of the order less than  $10^{-2}$ . On the contrary in the case of S2HDM, this effect could be larger [27].

Third, although in T2HDM the non-zero complex elements in  $\Sigma$  matrix can lead to FCNC, its effect is constrained only in up-type quarks, there is no FCNC between down-type quarks. As a result,  $K^0 - \bar{K}^0$  and  $B^0 - \bar{B}^0$  mixings can not be seriously modified by this model. But they could receive contributions in the S2HDM.

Let us investigate their new physics effects on the determination of angles  $\alpha, \beta, \gamma$ , respectively .

The 'gold-plated' channel for determining angle  $\beta$  is decay  $B_d^0 \rightarrow J/\psi K_S$ , which is dominated by tree level  $b \rightarrow c$  process. In both S2HDM and T2HDM, a considerable contribution from decay amplitude can arise from  $b \rightarrow c$  transition [11,26] . The reason is that the off-diagonal element  $\Sigma_{32}$  can relatively large in T2HDM. Moreover one can see from equation(3.3) that the CKM matrix elements associated with  $\Sigma_{32}$  is  $V_{tb}$  rather than  $V_{cb}$ . This can contribute to an additional enhancement factor of  $|V_{tb}/V_{cb}| \approx 25$ . The effective Lagrangian at tree level  $b \rightarrow c\bar{c}s$  has the form:

$$L_{eff} = -2\sqrt{2}G_F V_{cb} V_{cs}^* \left[ \bar{c}_L \gamma_\mu b_L \bar{s}_L \gamma^\mu c_L + 2\zeta e^{i\delta} \bar{c}_R b_L \bar{s}_L c_R \right], \quad (3.8)$$

where

$$\zeta e^{i\delta} = \begin{cases} (1/2)(V_{tb}/V_{cb})(m_c \tan \beta/m_H)^2 \xi^* & \text{for T2HDM} \\ (1/2)(V_{tb}/V_{cb})(\mu_{32}^{u\dagger}/m_H)^2 & \text{for S2HDM} \end{cases} \quad (3.9)$$

Using the formalism in references [11,26]. The decay amplitude of  $B \rightarrow J/\psi K_S$  can be written as  $A = A_{SM}[1 - \zeta e^{-i\delta}]$ , where  $A_{SM}$  denotes the amplitude in SM. If factorization holds there is no relative strong phase between  $W$ -and Charged-Higgs exchange process. So  $B$  and  $\bar{B}$  decay only differ by a CP-violating phase. Their ratio is given by:  $\bar{A}/A = (\bar{A}_{SM})/A_{SM} e^{-2i\phi_A}$ , where  $\phi_A = \tan^{-1}(\zeta \sin \delta/(1 - \zeta \cos \delta))$  is the correction to SM from charged Higgs exchange. Thus the time dependent asymmetry will measure  $\beta_{SM} + \phi_A$  rather than  $\beta_{SM}$

In the case of S2HDM, the new phase from amplitude can be obtained by simply replace the expression of  $\zeta e^{i\delta}$  in T2HDM in equation(3.9). However, the situation here is more complicate since there are additional contributions to  $B^0 - \bar{B}^0$  mixing, which will largely change the value of angle  $\beta$ . The new contributions come from the couplings  $\xi_i$  and  $\mu_{ij}$ . They are in general complex. Although the value of  $\mu_{ij}$  can be constrained from the measurement in  $x_d \equiv \Delta m_B/\Gamma_B$  [28–30], due to the large uncertainties of  $|V_{td}|$ ,  $\phi_{mix}^B$  can still be rather large [30].

The decay  $B_d^0 \rightarrow \pi^+ \pi^-$  is thought to be a good channel for the extraction of angle  $\alpha$ . Since  $B_d^0 \rightarrow \pi^+ \pi^-$  decay is dominated by  $b \rightarrow u(\bar{u}d)$  tree level process. The additional contribution from charged Higgs boson is proportional to the  $d$ -quark mass  $m_d$  which is negligibly small. There are also contributions from charged Higgs loop in penguin diagrams. By using the isospin analysis [12], those effects can be eliminated. Thus there are no additional new phases in  $B_d^0 \rightarrow \pi^+ \pi^-$  decay amplitude. In T2HDM, due to the absence of new phase in  $B^0 - \bar{B}^0$  mixing, this measurement of angle  $\alpha$  will give  $\alpha_{exp} = \alpha_{SM}$ , however in the S2HDM, as has been discussed in the previous section, it will give  $\alpha_{exp} = \alpha_{SM} - \phi_{mix}^B$ .

Finally let us consider the measurements of angle  $\gamma$ . As being proposed in [17] ,  $\gamma$  can be extracted from charged  $B$  decay  $B^\pm \rightarrow DK^\pm$ . By an independent measurement of six amplitude  $B^+ \rightarrow D^0 K^+, B^+ \rightarrow \bar{D}^0 K^+, B^+ \rightarrow D_{CP}^0 K^+, B^- \rightarrow D^0 K^-, B^- \rightarrow \bar{D}^0 K^-, B^- \rightarrow D_{CP}^0 K^-$  the angle  $\gamma$  can be in principle determined. Where  $D_{CP}^0 = \frac{\sqrt{2}}{2}(D^0 + \bar{D}^0)$  is the CP even eigenstate. In the SM the relations of these six amplitudes are :

$$\sqrt{2}A(B^+ \rightarrow D_1^0 K^+) = A(B^+ \rightarrow D^0 K^+) + A(B^+ \rightarrow \bar{D}^0 K^+), \quad (3.10)$$

$$\sqrt{2}A(B^- \rightarrow D_1^0 K^-) = A(B^- \rightarrow D^0 K^-) + A(B^- \rightarrow \bar{D}^0 K^-), \quad (3.11)$$

and

$$A(B^+ \rightarrow \bar{D}^0 K^+) = A(B^- \rightarrow D^0 K^-), \quad (3.12)$$

$$A(B^+ \rightarrow D^0 K^+) = A(B^- \rightarrow \bar{D}^0 K^-). \quad (3.13)$$

with  $|A(B^+ \rightarrow D_1^0 K^+)| \neq |A(B^- \rightarrow D_1^0 K^-)|$  these relations are illustrated in Fig. 1. If the magnitudes of the amplitudes can be measured experimentally, one can then extract the angle  $\gamma$ .

In both S2HDM and T2HDM, only  $b \rightarrow c$  process could be modified considerably by charged Higgs exchange. This implies that the two amplitudes  $A(B^+ \rightarrow D^0 K^+)$  and  $A(B^+ \rightarrow \bar{D}^0 K^+)$  which are dominated by the  $b \rightarrow u$  transitions will remain unchanged and the angle between them is still given by  $2\gamma$ . However, the new phase can contribute to  $A(B^+ \rightarrow \bar{D}^0 K^+)$  and  $A(B^+ \rightarrow D^0 K^+)$  since they are dominated by the  $b \rightarrow c$  at tree level subprocess. The relation of (3.12) will be modified to be

$$A(B^+ \rightarrow \bar{D}^0 K^+) = e^{2i\phi_A} A(B^- \rightarrow D^0 K^-) \quad (3.14)$$

As it is shown in Fig.1, if  $\phi_A$  can be extracted from equation(2.8), then the angle  $\gamma$  can be obtained by measuring those six amplitudes. The angle  $\gamma$  determined in this way is equal to the one in the SM, i.e.  $\gamma_{exp} = \gamma_{SM}$ .

In summary, the  $B_d^0 \rightarrow J/\psi K_S$  decay can be affected in both models. In the T2HDM, it is affected through the  $B$  decay amplitudes. In the S2HDM, the new phase could arise from both  $B^0$  mixing and decay amplitudes. In the extraction of  $\alpha$ , the effect of T2HDM is negligible, but the one of S2HD can contribute a new phase from decay amplitude through  $B^0$  mixing. In the extraction of  $\gamma$ , if the new phase from decay amplitude can be determined from equation(2.8), the method of measuring  $\gamma$  by combining the six amplitudes of  $B^\pm \rightarrow DK^\pm$  still works and  $\gamma_{exp}$  will be equal to  $\gamma_{SM}$ .

Since  $\beta_{SM}$  can be extracted through  $K \rightarrow \pi \nu \bar{\nu}$ , it is then possible to extract  $\phi_{mix}^B$  from (2.2) or (2.3). If  $\phi_{mix}^B \neq 0$  is observed in the future experiment, it implies that the T2HDM is disfavorable.

#### IV. CONCLUSIONS

In conclusion, the distinguishment of different new physics models is discussed. It has been seen that by comparing their different behaviors in  $B^0 - \bar{B}^0$  mixing and  $B$  decay amplitudes, these models can be partially separated. The distinguishments between T2HDM and S2HDM have been discussed in detail. The new physics effects in measuring UT angles  $\alpha, \beta$  and  $\gamma$  from these two models have been examined. It has been seen that by measuring  $\alpha, \beta, \gamma$  from  $B_d^0 \rightarrow \pi^+ \pi^-$ ,  $K \rightarrow \pi \nu \bar{\nu}$  and  $B^\pm \rightarrow DK^\pm$  respectively, the new phase  $\phi_{mix}^{B_d}$  can be extracted. Since there is no contribution from  $B^0$  mixing in T2HDM, if  $\phi_{mix}^{B_d} \neq 0$  from the future experiment is founded, the T2HDM will be excluded. The situation will be different in the S2HDM where a non-zero value of  $\phi_{mix}^{B_d}$  is allowed. This is because the T2HDM can be regarded as one of the special cases of S2HDM.

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## FIGURES

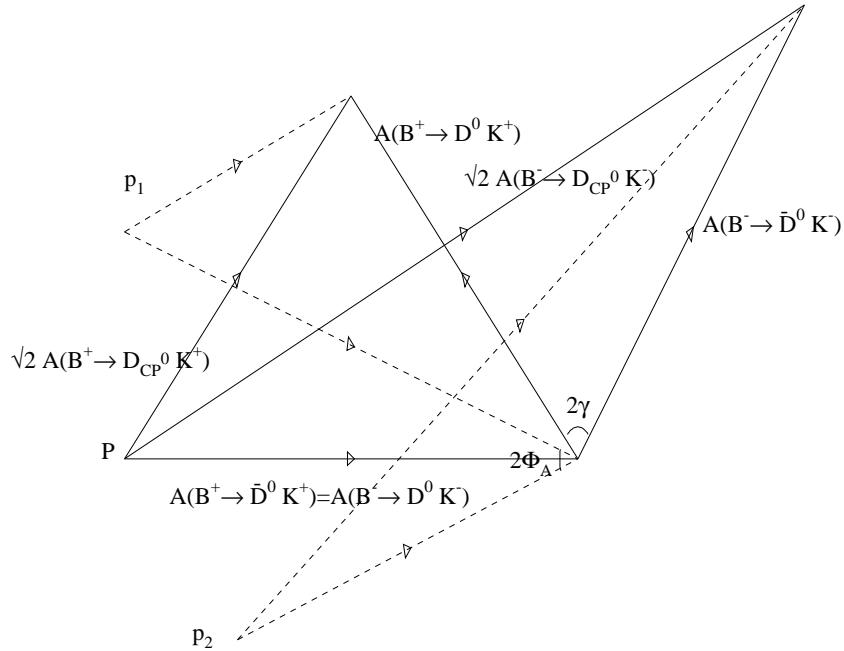


FIG. 1. The triangle relations of six decay amplitudes  $B^\pm \rightarrow D^0 K^\pm$ ,  $B^\pm \rightarrow \bar{D}^0 K^\pm$ , and  $B^\pm \rightarrow D_{CP}^0 K^\pm$ , in the SM (solid line) and in the models with only new phase in  $b \rightarrow c$  tree level transition such as S2HDM and T2HDM (dashed line). The relation  $A(B^+ \rightarrow \bar{D}^0 K^+) = A(B^- \rightarrow D^0 K^-)$  which holds in the SM is violated when new phase is involved. If the angle between them (denoted by  $2\phi_A$ ) can be determined experimentally, those triangles can still be used to extract angle  $\gamma$  and will be found to be equal to the one in the SM.